Newtonian Noise

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Overview

What is Newtonian noise?

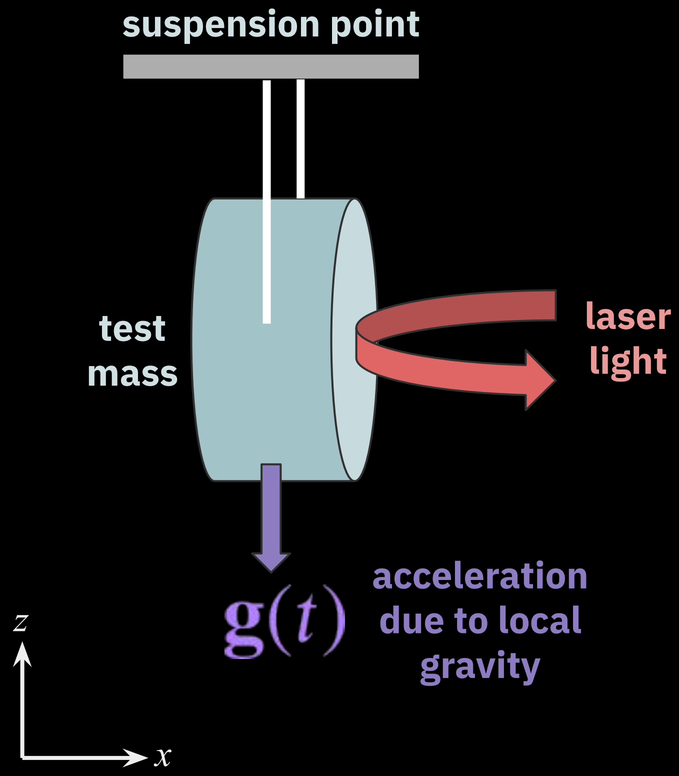
Seismic sources

Atmospheric sources

Mitigation schemes

Implementation in pygwinc

Newtonian noise (a.k.a. gravity gradient noise)



$$\mathbf{g}(t) = \mathbf{g}_0 + \delta\mathbf{g}(t)$$

$\mathbf{g}_0$: large, static component

$\delta\mathbf{g}(t)$: fluctuating component (Newtonian noise)

$$\ddot{h}(t) = \frac{\delta g_x(t)}{L} \Leftrightarrow h(\omega) = \frac{\delta g_x(\omega)}{-\omega^2 L}$$

Computing NN

Newton's law of universal gravitation:

$$\mathbf{g}(t) = G_\mathrm{N} \int \mathrm{d}^3\mathbf{r}\, \frac{\rho(\mathbf{r}, t)}{r^2}\, \hat{\mathbf{r}}$$

NN can be caused by

- the **motion of a point mass**,
- a **change in the density distribution in an extended mass**, or
- a **displacement of the boundary of an extended mass**, even in the absence of a change in density.

Harms (2019) *LRR* **22**(6)

What causes NN?

Continuous sources:

- Ambient seismic waves
- Ambient acoustic waves
- Other atmospheric processes
- Flowing or bubbling liquids

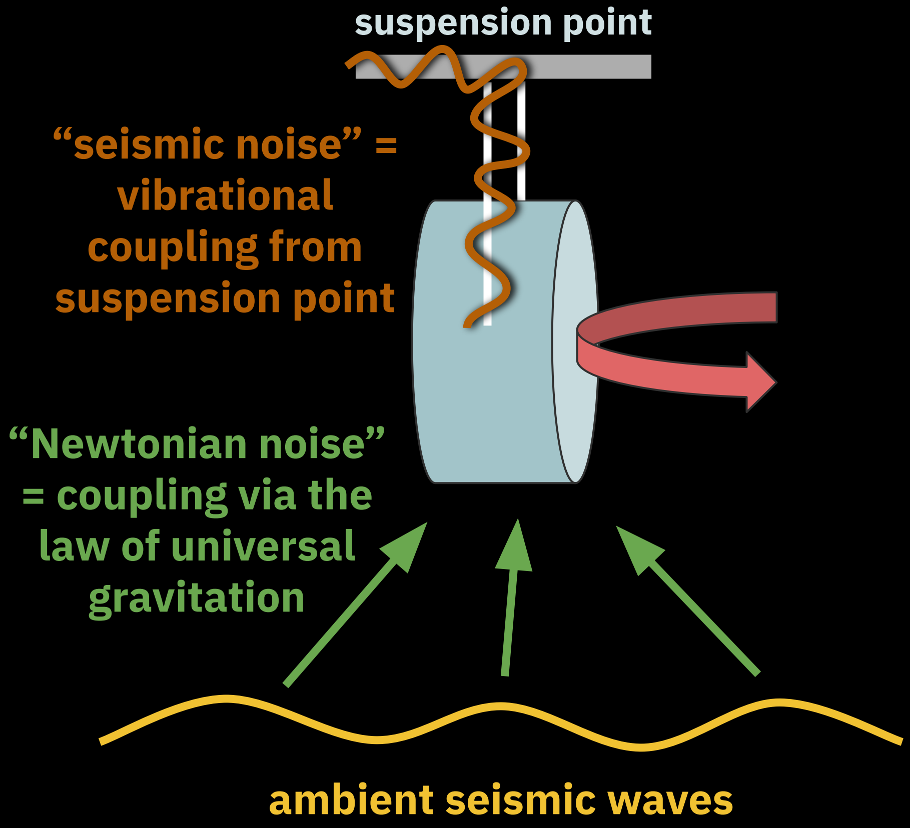
} implemented in pygwinc (Ambient seismic waves, Ambient acoustic waves)

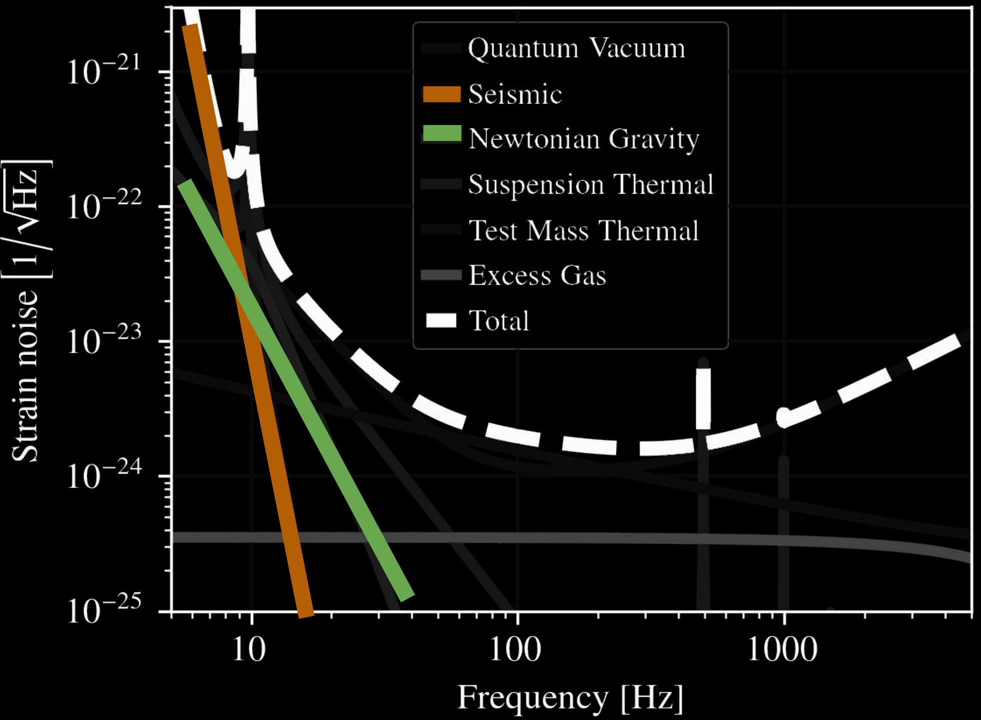
Transient sources:

- Earthquakes
- Shock waves
- Cars and airplanes
- Humans [Thorne & Winstein (1998) *PRD* **60** 082001]
- Tumbleweeds [Creighton (2008) *CQG* **25**(12)]

Seismic Newtonian noise

How is seismic NN different from “seismic noise”?





Ground model

Ground is modeled as a **linear elastic medium**; seismic waves are weak perturbations to the steady-state

The simplest possible ground model is a **homogeneous, isotropic, infinite half-space**, which is mechanically characterized by only three numbers:

- Bulk modulus, K: stiffness in response to pressure — typically ~1 MPa (soils) to >10 GPa (rocks)
- Shear modulus, μ: stiffness in response to shear — typically ~1 MPa (soils) to >10 GPa (rocks)
- Density, ρ_0: typically several thousand kg/m^3

Seismic waves

Body waves: travel through the bulk of the ground

- Pressure (P) waves
- Shear (S) waves — two polarizations

Surface waves:

- Rayleigh waves
- (Love waves — do not appear in homogeneous media, and don't make NN in a half-space model)

Waves described by a displacement field $\boldsymbol{\xi}(\mathbf{r},t)$, which is connected to density fluctuations via $\delta\rho(\mathbf{r},t) = -\nabla\cdot[\rho_0(\mathbf{r},t)\boldsymbol{\xi}(\mathbf{r},t)]$

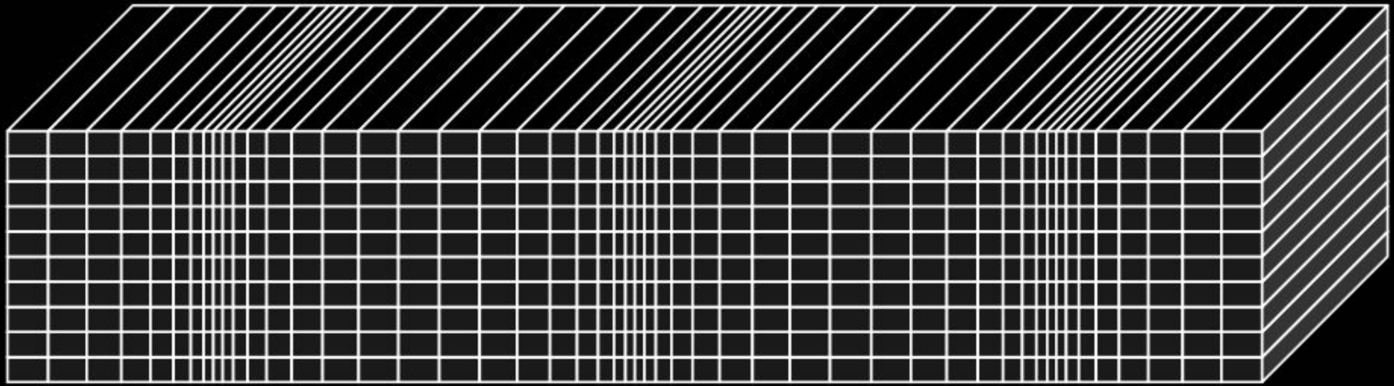
Seismic P-waves

Longitudinal displacement field: $\boldsymbol{\xi}^{(P)}(\mathbf{r},t) = \xi_0^{(P)} \cos(\mathbf{k}^{(P)} \cdot \mathbf{r} - \omega t)\, \hat{\mathbf{k}}$

Creates NN primarily through density perturbation

Wave speed:

$$c_P = \sqrt{\frac{K + 4\mu/3}{\rho_0}}$$



Shearer (2009), *Introduction to Seismology*, Cambridge

Seismic S-waves

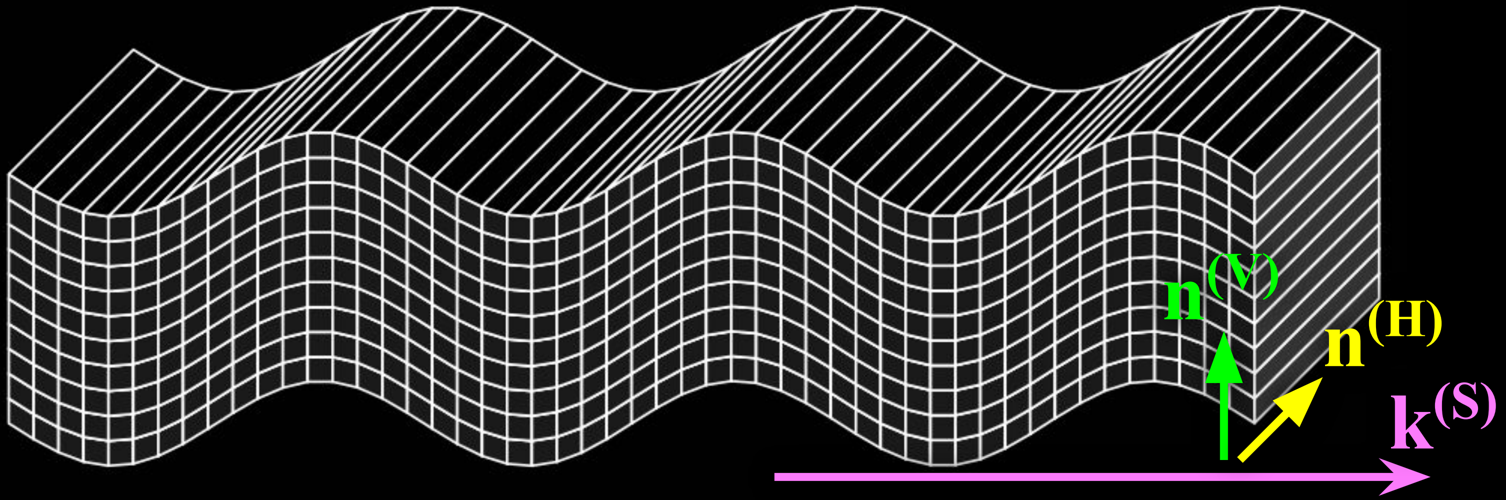
Transverse displacement field, two polarizations:

$$\xi^{(SH)}(\mathbf{r}, t) = \xi_0^{(SH)} \cos(\mathbf{k}^{(S)} \cdot \mathbf{r} - \omega t)\, \hat{\mathbf{n}}^{(H)}$$

$$\xi^{(SV)}(\mathbf{r}, t) = \xi_0^{(SV)} \cos(\mathbf{k}^{(S)} \cdot \mathbf{r} - \omega t)\, \hat{\mathbf{n}}^{(V)}$$

Motion is pure shear (no density changes). S-waves produce NN only by displacing boundaries.

Wave speed $c_S = \sqrt{\mu/\rho_0}$

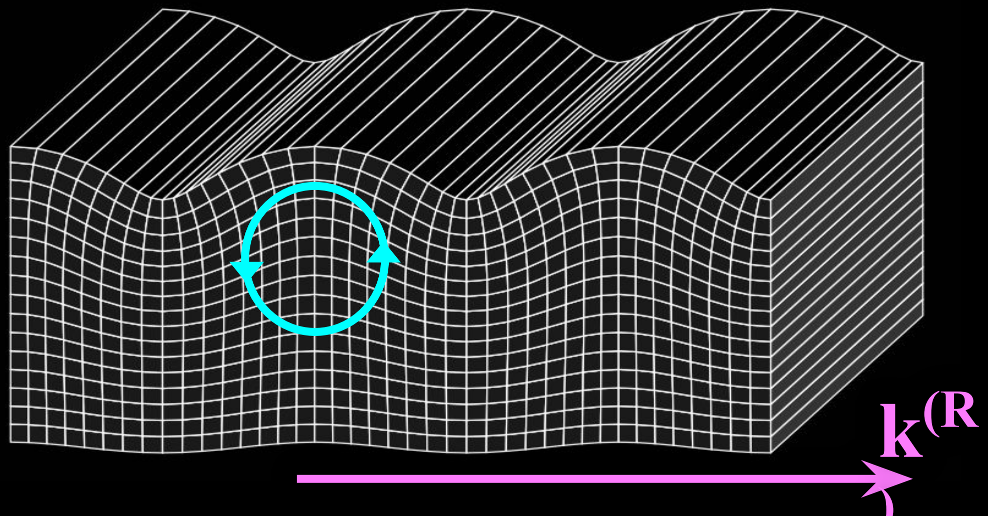


Seismic Rayleigh waves

Surface acoustic wave; displacement field decays exponentially with depth

Retrograde elliptical particle motion: **creates NN through both density perturbation and ground surface displacement**

Travel at ~90% the speed of S-waves.



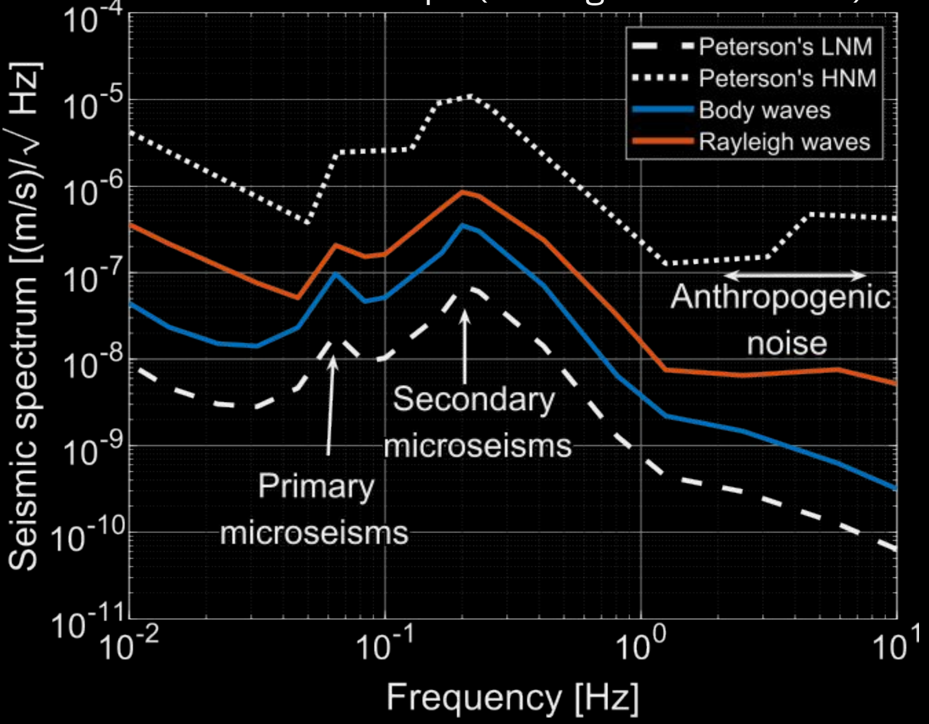
Estimating seismic Newtonian noise

1. Establish a model for the ground.
2. Write down the displacement field $\boldsymbol{\xi}(\mathbf{r},t)$ of each seismic wave.
3. Compute the corresponding density fluctuation and boundary perturbation.
4. Compute the local gravity fluctuation $\delta\mathbf{g}(t)$ due to each wave, according to Newton's law of universal gravitation.
5. Account for the full seismic field by summing over the superposition of many individual seismic waves.

   We typically start with an assumption of a **homogeneous, isotropic** seismic field.
   In any case, we need to assume something about the body- and surface-wave content of the field…

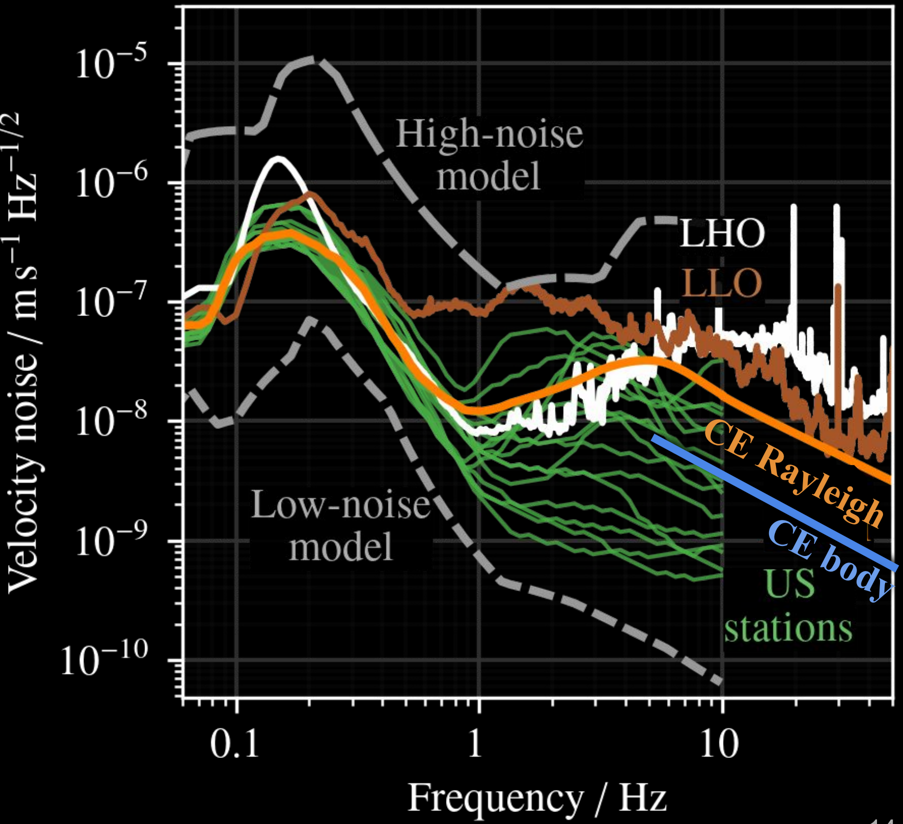
Seismic motion

Einstein Telescope (underground detector)



Amann et al. (2020) *RSI* **91**, 094504

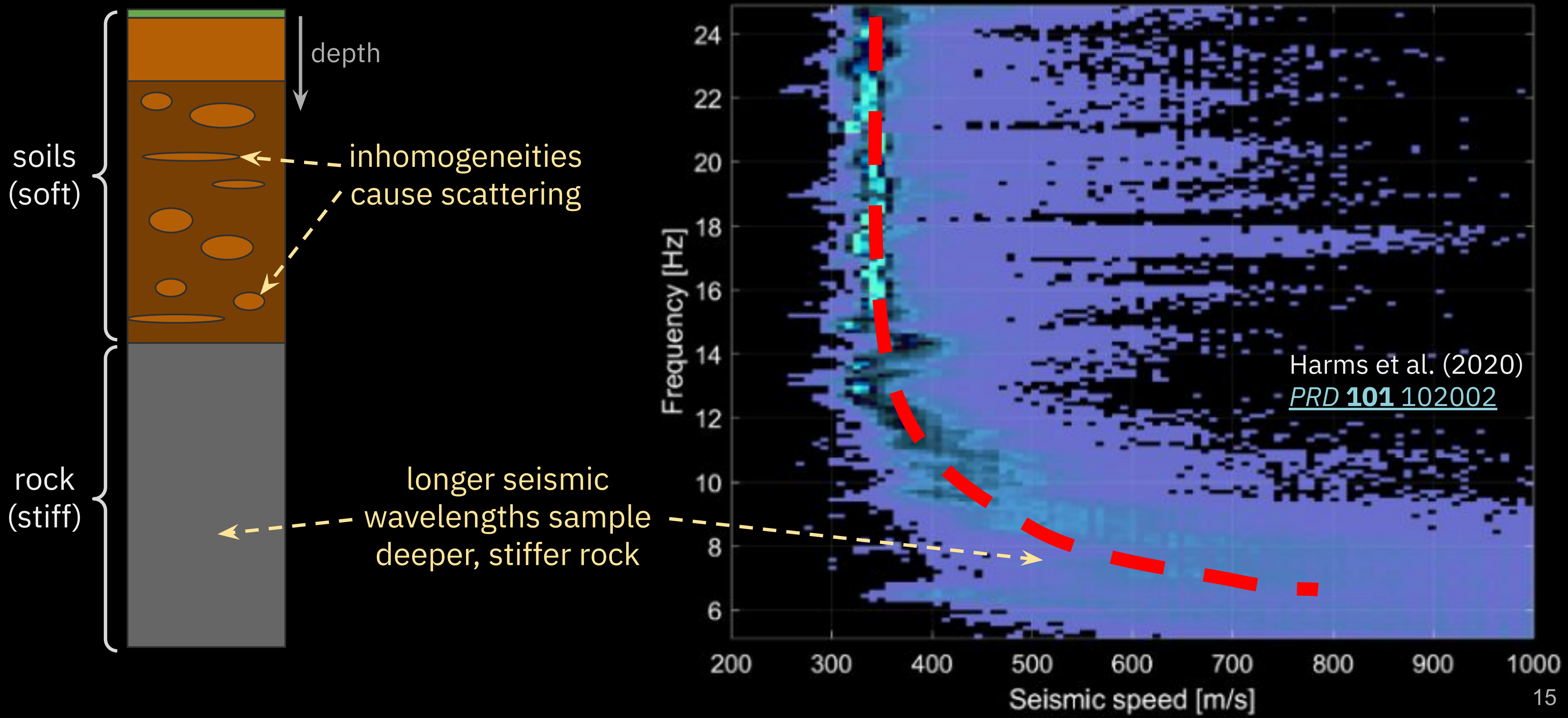
LIGO & CE (surface detectors)



Hall et al. (2021) arXiv:2012.03608

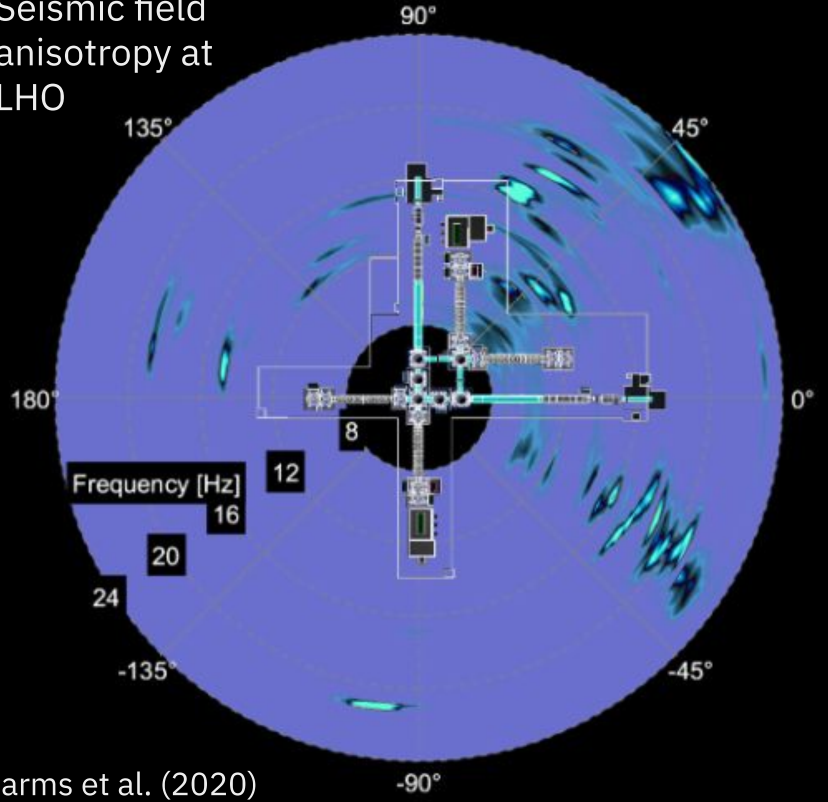
Beyond the homogenous, isotropic ground model...

Rayleigh-wave dispersion at LHO



Beyond the homogeneous, isotropic seismic field...

Seismic field anisotropy at LHO



Harms et al. (2020) *PRD* **101** 102002

Seismic fields (at least on the surface) are not isotropic.

In current detectors, most of the seismic noise is generated by the GW facility itself, sometimes very close to the test masses.

Mitigation: site selection and detector geometry

0. Go into space
1. Pick a low-noise site

   Can be 100× variation in typical seismic amplitude at 10 Hz between sites
2. Keep the site low noise

   Isolate vibrating machinery from the ground near the test masses
3. Make the detector longer

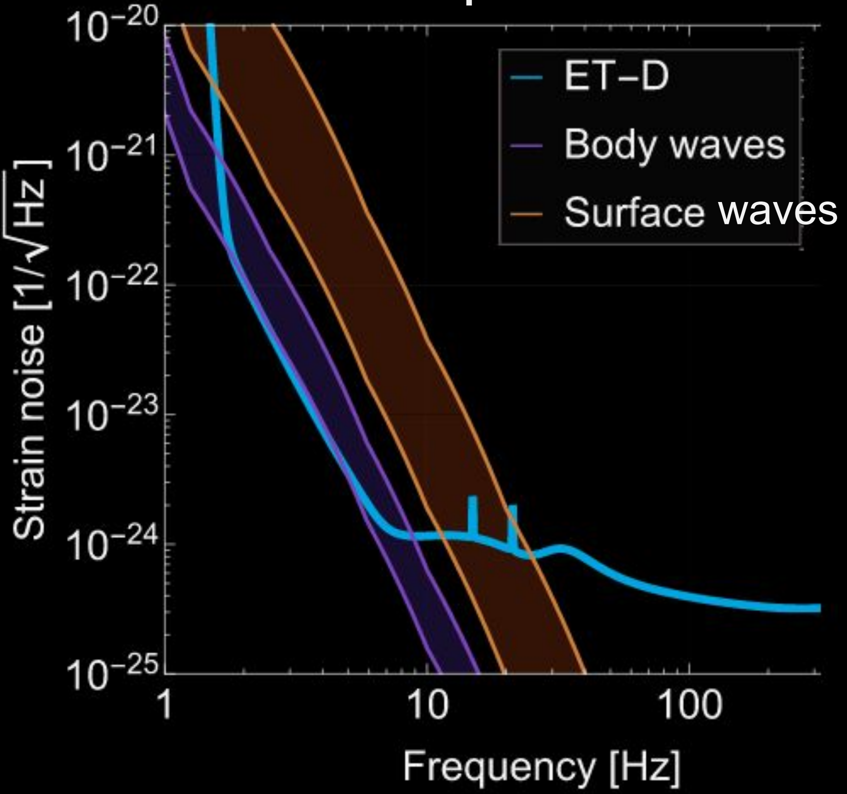
   Equivalent strain amplitude from NN scales like $1/L$.
4. Go underground

   Surface motion dominated by Rayleigh waves, which decay exponentially with depth.
5. If on the surface, place test masses further away from the ground

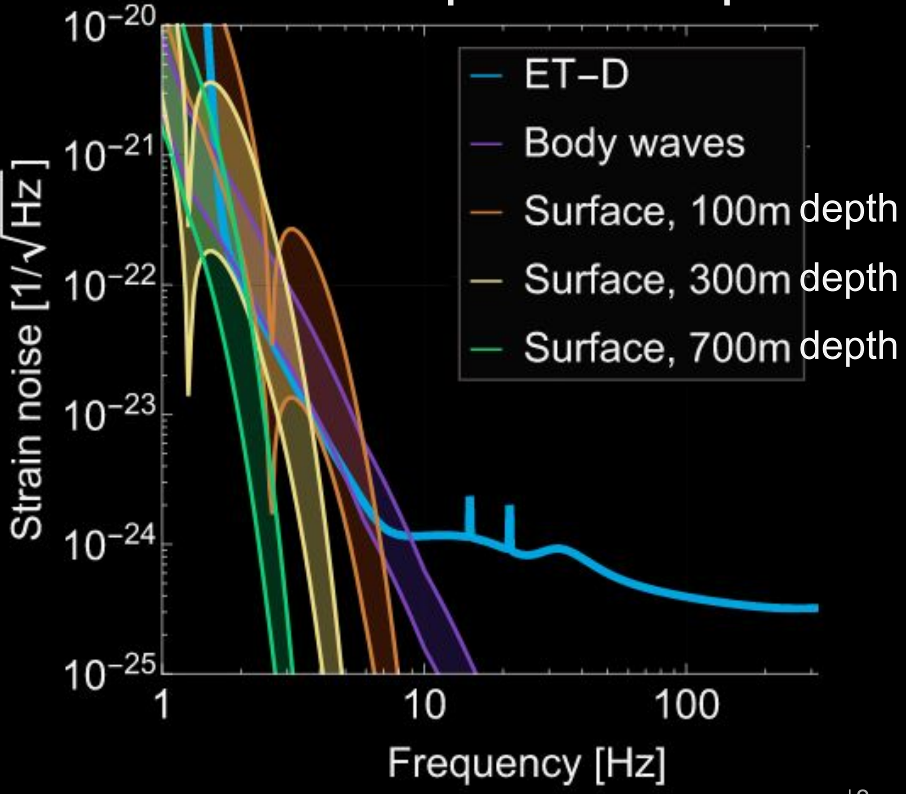
   Ditches, trenches, basements, geofoam...

Mitigation: going underground

Einstein Telescope on Earth's surface

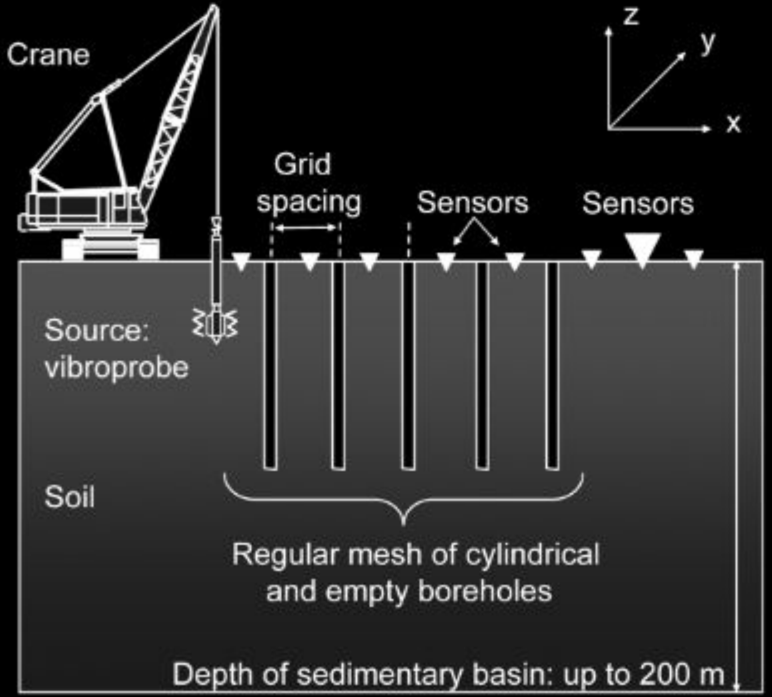


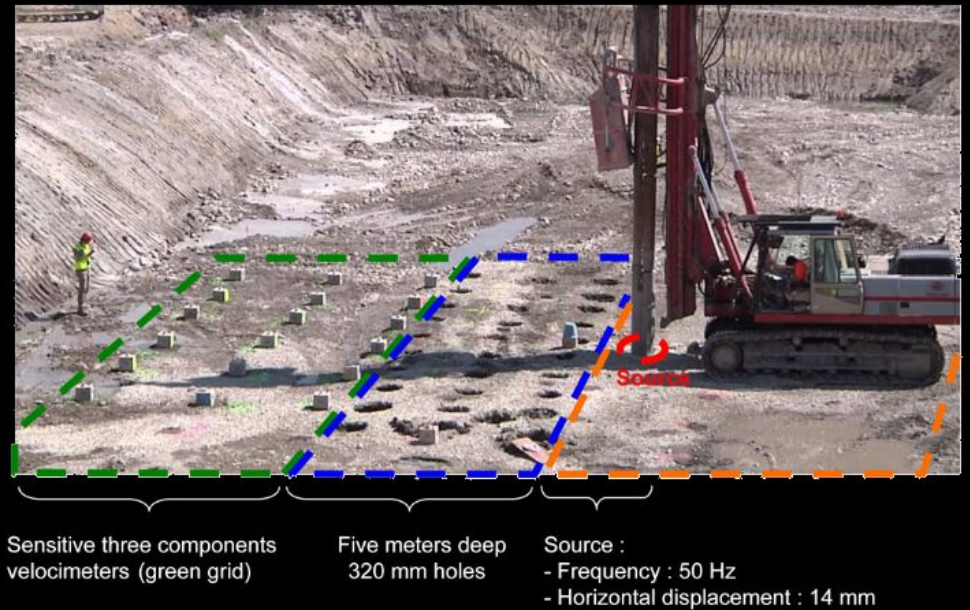
Einstein Telescope at several depths



Badaracco & Harms (2019), *CQG* **36** 145006

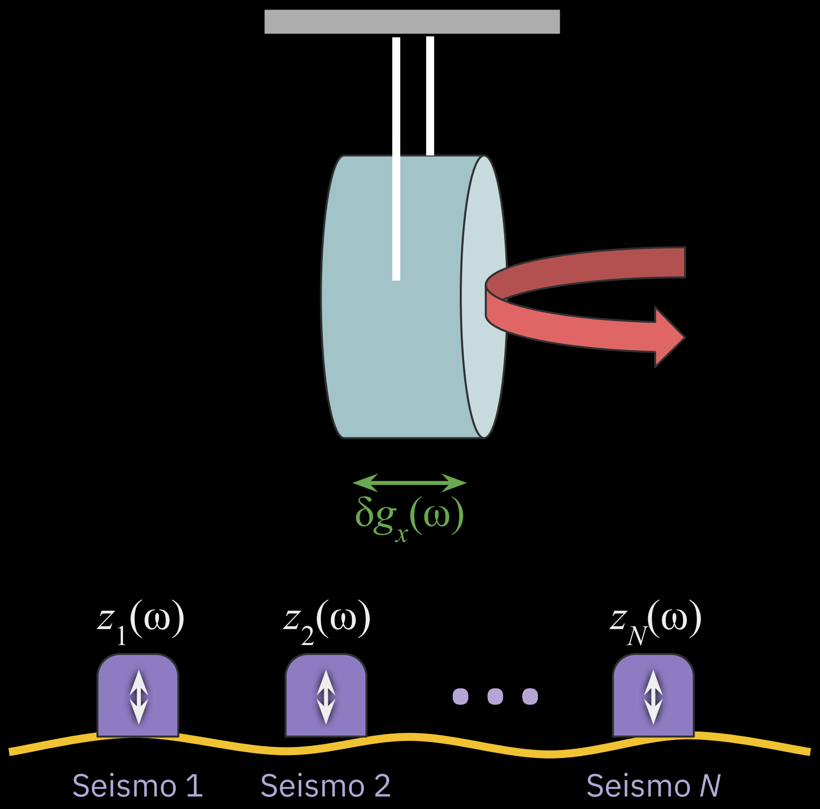
Mitigation: seismic metamaterials





Brûlé et al. (2014) *PRD* **112** 133901

Mitigation: seismometer array subtraction



Goal: use a network of seismometers to estimate the local gravity fluctuation at the test mass.

Often formulated as a **Wiener filter problem**: given *N* seismometer data records $\{z_i(\omega)\}$, find a set of filters $\{W_i(\omega)\}$ such that the sum $\sum W_i(\omega) z_i(\omega)$ is an optimal estimate of the local gravity $\delta g_x(\omega)$ at the test mass.

"Optimal" here means minimizing the mean-squared error between the estimate and the true value of $\delta g_x(\omega)$.

Beker et al. (2011) *Gen Relativ Gravit* **43**, 623–656

Mitigation: seismic array subtraction (2)

Wiener filter solution requires measuring (or modeling):

1. the *N* cross-spectra $\{H_i(\omega)\}$ from each seismometer to the test mass local gravity,
2. and the *N*×*N* cross-spectra $\{C_{ij}(\omega)\}$ between all the seismometers.

Then the optimal filters are $W_i = H_j(C^{-1})_{ij}$, and the residual mean-squared error is

$$R(\omega) = 1 - \frac{\mathbf{H}(\omega)^{\top}\mathrm{C}(\omega)^{-1}\mathbf{H}(\omega)}{S_{\delta g}(\omega)}$$

where $S_{\delta g}(\omega)$ is the power spectrum of the Newtonian noise in the detector.

Mitigation: seismic array subtraction (3)

Noise limit: $R(\omega) > 1/(N \cdot \mathrm{SNR}(\omega)^2)$

- Typical seismometer SNR ~ 100 to 1000
- Typical array size of order 10

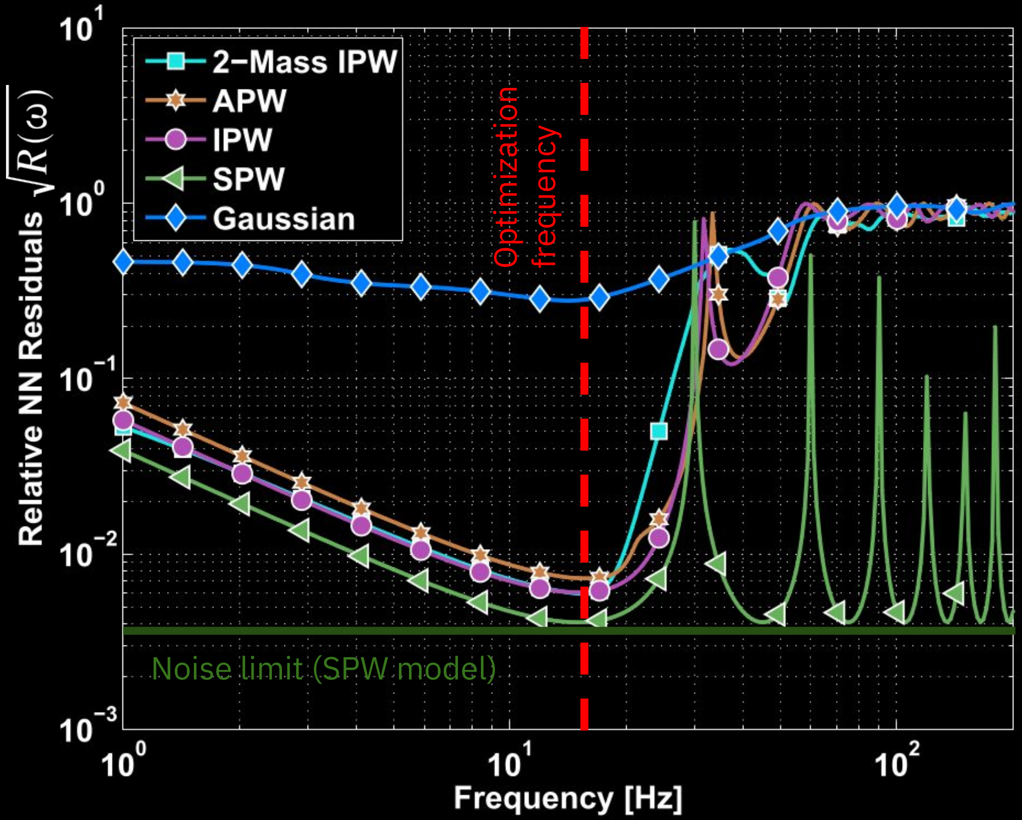
Geometric limit: depends on the seismometer coordinates and the seismic field

- Array scale should match the seismic wavelength of interest
- Array coordinates must be optimized, generally numerically

  Driggers et al. (2012) *PRD* **86** 102001

$$R(\omega) = 1 - \frac{\mathbf{H}(\omega)^{\top} \mathrm{C}(\omega)^{-1} \mathbf{H}(\omega)}{S_{\delta g}(\omega)}$$

Example: aLIGO Rayleigh-wave NN cancellation model



Coughlin et al. (2016) *CQG* **33** 24401

In simulation, 100× cancellation of Rayleigh waves seems feasible on the surface, over a limited frequency band.

Also in simulation, 10× cancellation of body waves seems feasible underground. [Badaracco & Harms (2019), *CQG* **36** 145006]

(A more cautious assumption might be 10× Rayleigh wave cancellation and 3× body wave cancellation…)

Seismic NN: what is computed in pygwinc?

```
newtonian.gravg_rayleigh
newtonian.gravg_pwave
newtonian.gravg_swave
```

Assumptions:

1. Ground is a homogenous, isotropic, infinite half-space
2. Seismic field is a stationary, homogeneous, isotropic superposition of plane waves arriving from all directions (2π sr for P- and S-waves; 2π rad for Rayleigh waves)
3. For Rayleigh waves, the ground motion is an ad-hoc broken power law with an optional constant describing the amount of subtraction
4. For body waves, the ground motion is specified as a multiple of the Peterson low-noise model
5. Scattering/interconversion of seismic waves is ignored

Example (from Cosmic Explorer 2)

In the `ifo.yaml` file...

```
Seismic:
  Site: 'LHO'                     # LHO or LLO (only used for Newtonian noise)
  KneeFrequency: 5                # Hz; freq where 'flat' noise rolls off
  LowFrequencyLevel: 1e-9         # m/rtHz; seismic noise level below f_knee
  KneeFrequencyHorizontal: 4      # Hz; freq where 'flat' noise rolls off
  LowFrequencyLevelHorizontal: 1e-9 # m/rtHz; seismic noise level below f_knee
  Gamma: 0.8                      # abruptness of change at f_knee
  Rho: 1.8e3                      # kg/m^3; density of the ground nearby
  Beta: 0.8                       # quiet times beta: 0.35-0.60
                                  # noisy times beta: 0.15-1.4
  Omicron: 10                     # Feedforward cancellation factor
  TestMassHeight: 1.5             # m
  pWaveSpeed: 600                 # m/s
  sWaveSpeed: 300                 # m/s
  RayleighWaveSpeed: 250          # m/s
  pWaveLevel: 15                  # Multiple of the Peterson NLNM amplitude
  sWaveLevel: 15                  # Multiple of the Peterson NLNM amplitude
  PlatformMotion: '6D'
```

ground motion model used by gravg_rayleigh

ground motion model used by gravg_pwave and gravg_swave

Atmospheric Newtonian noise

Infrasound NN

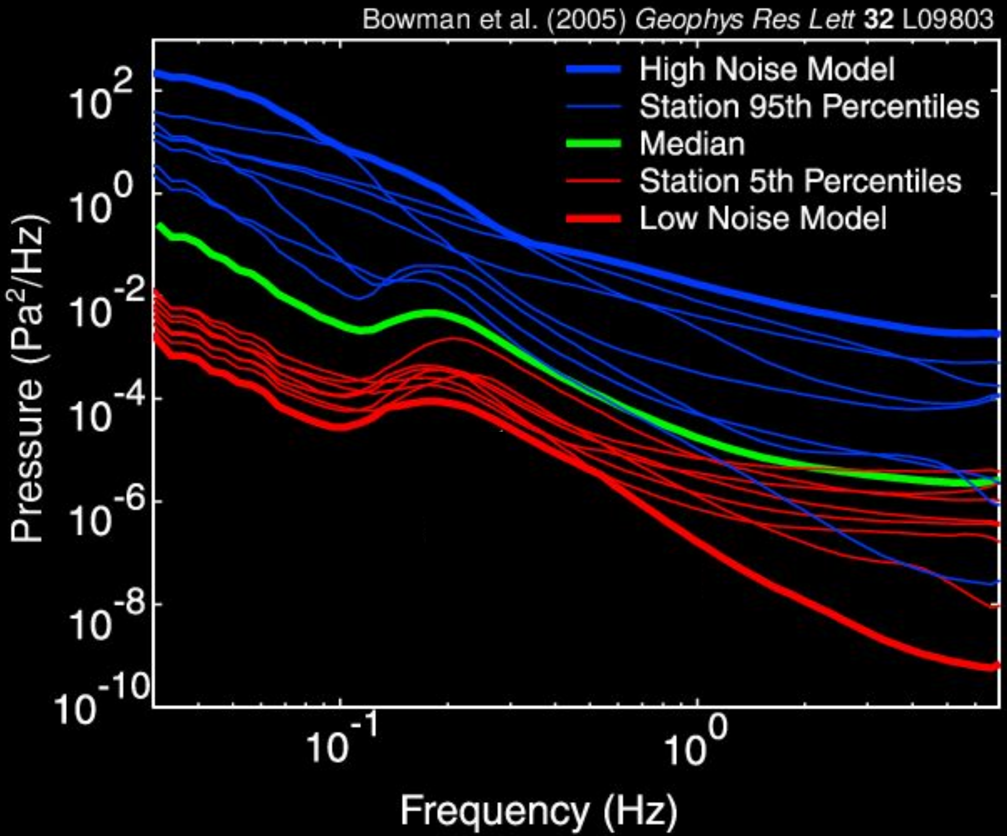
Sound (acoustic waves) below 20 Hz

Pressure field $p(\mathbf{r},t) = p_0\cos(\mathbf{k}\cdot\mathbf{r} - \omega t)$; speed $c_{\text{sound}} = 340$ m/s

Produces density fluctuation via $\gamma(\delta\rho/\rho) = (\delta p/p)$; $\gamma = 1.4$ for air

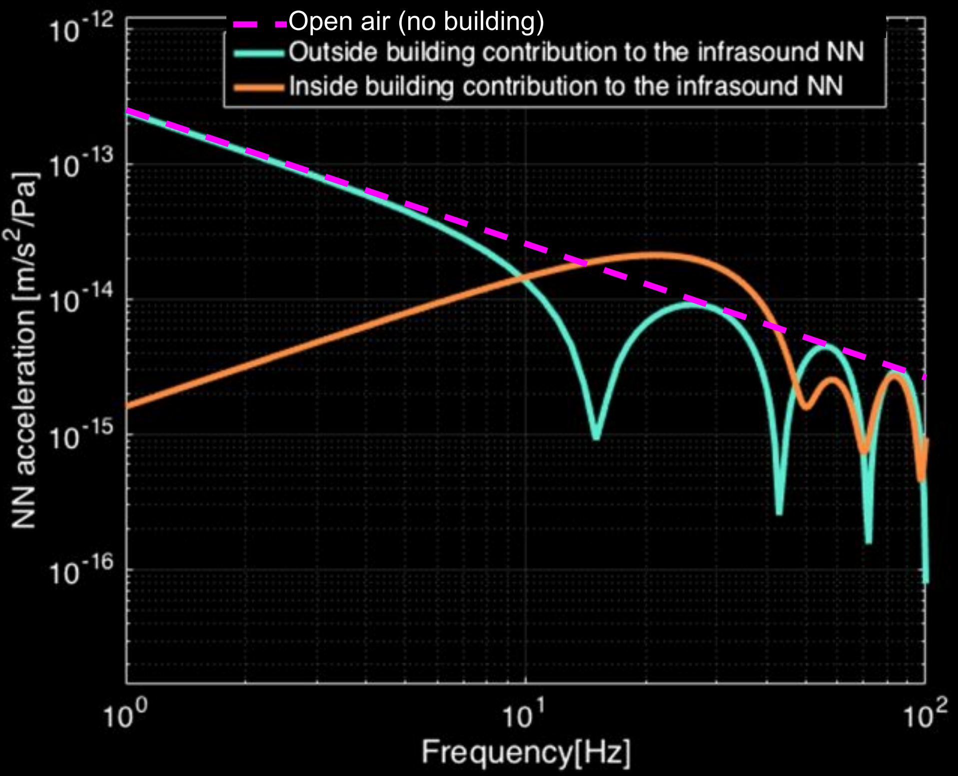
NN calculations often assume a homogeneous, isotropic acoustic wave field, but real wavefields can be anisotropic.

Infrasound spectrum



Median model suggests 1 mPa/Hz$^{1/2}$ above a few hertz, but potentially limited by sensor noise or calibration.

Structural mitigation of infrasonic NN?



Fiorucci et al. (2018) *PRD* **97** 062003

An exponential suppression of outside infrasound NN requires going underground: at a depth d, the cutoff frequency is $f = c_{\text{sound}}/d$

Buildings and burial do not save you from noise generated from indoor detector infrastructure, like HVAC systems.

Subtracting infrasonic NN?

Navier–Stokes equations are highly nonlinear, leading to contributions to the pressure field from **turbulent flow**, particularly during windy times:

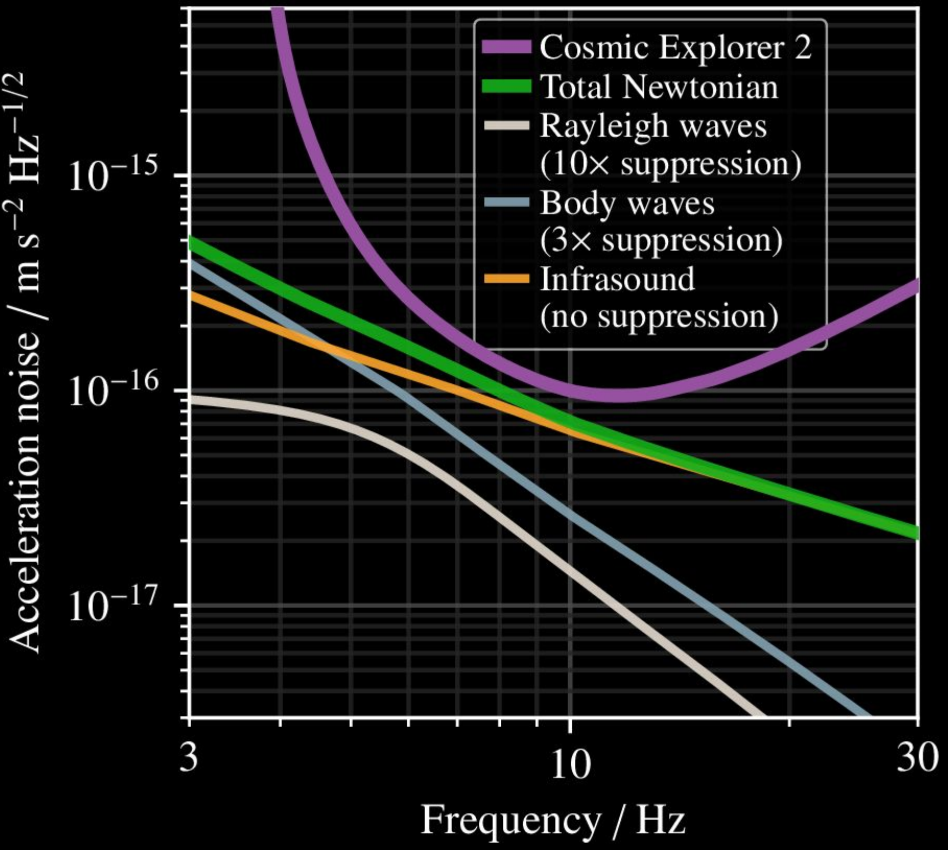
- Intrinsic wind noise (turbulence–turbulence and turbulence–shear interactions)
- Stagnation pressure: wind blowing against the sensor.

Raspet et al. (2019) in Infrasound Monitoring for Atmospheric Studies (Le Pichon et al., eds.), Springer.

LIDAR? Currently can sense $\delta p/p \sim 10^{-3}$, but infrasound NN fluctuations are $\sim 10^{-7}$.

Fiorucci et al. (2018) *PRD* **97** 062003

NN in a future surface detector



Assumed 1 mPa/Hz$^{1/2}$ pressure spectrum; no consideration of building.

Other atmospheric NN sources

Inhomogeneities of air temperature and humidity, which are advected past the test mass by wind:

- Exponentially suppressed above a few hertz for typical surface detector parameters (wind speeds ~10 m/s, building size ~10 m)

Aeroacoustic noise (Lighthill noise):

- Pressure fluctuations sourced by turbulent flow
- Also unlikely to be significant above a few hertz

Harms (2019) *LRR* **22**(6)

What is computed in pygwinc?

`newtonian.atmois` (infrasonic NN)

Assumptions:

- Stationary, homogeneous, isotropic superposition of plane waves
- Pressure field can be specified as a power law, or else as the global median (Bowman) model
- Ground is perfectly reflecting (no seismic/infrasound interconversion)

Example (from Cosmic Explorer 2)

In the `ifo.yaml` file...

```yaml
Atmospheric:
  AirPressure: 101325                 # Pa
  AirDensity: 1.225                   # kg/m**3
  AirKinematicViscosity: 1.8e-5       # m**2/s
  AdiabaticIndex: 1.4                 #
  SoundSpeed: 344                     # m/s
  WindSpeed: 5                        # m/s; typical value
  Temperature: 300                    # K
  TempStructConst: 0.2                # K**2/m**(2/3);
  TempStructExp: 0.667                #
  TurbOuterScale: 100                 # m
  # TurbEnergyDissRate: 0.01          # m**2/s**3
  KolmEnergy1m: 1                     # Kolmogorov energy spectrum at 1/m [m**2/s**2]
```

Homework

Jan Harms (2019), "Terrestrial gravity fluctuations", *Living Reviews in Relativity* **22**(6), https://doi.org/10.1007/s41114-019-0022-2

Read through 3.2.1.:

- Understand how to formulate Newton's law in terms of gravitational potential, and how to separate potential fluctuations into to bulk and surface contributions.

Read through 3.4.2.:

- Familiarize yourself with the displacement fields for Rayleigh waves (Eq. 91, referring back to Eqs. 37–41 and related discussion as necessary).
- Work through the subsequent calculation for the gravitational potential fluctuations for a Rayleigh wave.