Gravitational Wave Transients

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Build-a-detector workshop

What is a "gravitational wave transient"

- No universally accepted definition.
- Let's go for:

  > *"Any gravitational wave signal of duration less than* ~ 1 *day, excluding signals from the inspiral/merger/ring-down of compact binary coalescence"*

Examples LSC/LVC/LVK papers on transients

Of $\sim$ 210 published LSC/LVC/LVK papers, approx 47 target transients. Examples:

1. “Optically targeted search for gravitational waves emitted by core-collapse supernovae during the first and second observing runs of advanced LIGO and advanced Virgo” (2020).
2. “Search for Gravitational-wave Signals Associated with Gamma-Ray Bursts during the Second Observing Run of Advanced LIGO and Advanced Virgo” (2019).
3. “All-sky search for short gravitational-wave bursts in the second Advanced LIGO and Advanced Virgo run” (2019).
4. “All-sky search for long-duration gravitational-wave transients in the second Advanced LIGO observing run” (2019).
5. “Search for Gravitational Waves from a Long-lived Remnant of the Binary Neutron Star Merger GW170817” (2019).
6. “Search for Transient Gravitational-wave Signals Associated with Magnetar Bursts during Advanced LIGO’s Second Observing Run” (2019).
7. “Constraints on cosmic strings using data from the first Advanced LIGO observing run” (2018).
8. “Search for transient gravitational waves in coincidence with short-duration radio transients during 2007-2013” (2016).
9. “Multimessenger search for sources of gravitational waves and high-energy neutrinos: Initial results for LIGO-Virgo and IceCube” (2014).
10. “Implications for the Origin of GRB 051103 from LIGO Observations” (2012).
11. “Search for gravitational waves associated with the August 2006 timing glitch of the Vela pulsar” (2011).

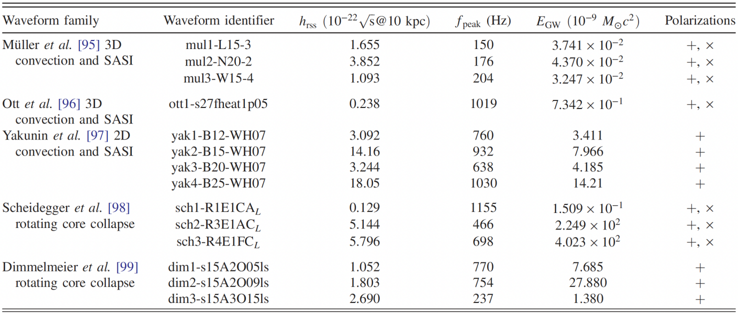
Types of transient

- Let's divide into three classes:
  - Catastrophic collapse
  - Cosmic strings
  - Oscillating/rotating neutron stars

  We'll mention all three, but focus on the third.

Stellar collapse

- Supernova explosion mechanism not well understood, but collapse will be asymmetric at some level.
- Most recent LVK paper targeted 5 optically observed SN, all quite close (7–20 Mpc).
- Predicted peak frequency for GW emission uncertain:



Waveform family	Waveform identifier	$h_{\rm rss}$ $(10^{-22}\sqrt{s}@10\ {\rm kpc})$	$f_{\rm peak}$ (Hz)	$E_{\rm GW}$ $(10^{-9}\ M_{\odot}c^2)$	Polarizations
Müller *et al.* [95] 3D convection and SASI	mul1-L15-3	1.655	150	3.741×10^{-2}	+, ×
	mul2-N20-2	3.852	176	4.370×10^{-2}	+, ×
	mul3-W15-4	1.093	204	3.247×10^{-2}	+, ×
Ott *et al.* [96] 3D convection and SASI	ott1-s27fheat1p05	0.238	1019	7.342×10^{-1}	+, ×
Yakunin *et al.* [97] 2D convection and SASI	yak1-B12-WH07	3.092	760	3.411	+
	yak2-B15-WH07	14.16	932	7.966	+
	yak3-B20-WH07	3.244	638	4.185	+
	yak4-B25-WH07	18.05	1030	14.21	+
Scheidegger *et al.* [98] rotating core collapse	sch1-R1E1CA$_L$	0.129	1155	1.509×10^{-1}	+, ×
	sch2-R3E1AC$_L$	5.144	466	2.249×10^{2}	+, ×
	sch3-R4E1FC$_L$	5.796	698	4.023×10^{2}	+, ×
Dimmelmeier *et al.* [99] rotating core collapse	dim1-s15A2O05ls	1.052	770	7.685	+
	dim2-s15A2O09ls	1.803	754	27.880	+
	dim3-s15A3O15ls	2.690	237	1.380	+

Table from Abbott+ PRD **101** 084002 (2020)

Stellar collapse

- Energetics suggest that we'd need an extremely energetic and nearby event to make a detection:

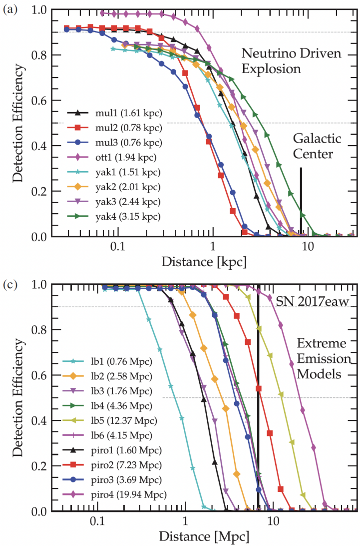


Table from Abbott+ PRD **101** 084002 (2020)

Cosmic strings

- Can have GW emission from cusps/kinks.
- Large range of GW frequencies relevant; most recent search covered full LIGO/Virgo band.

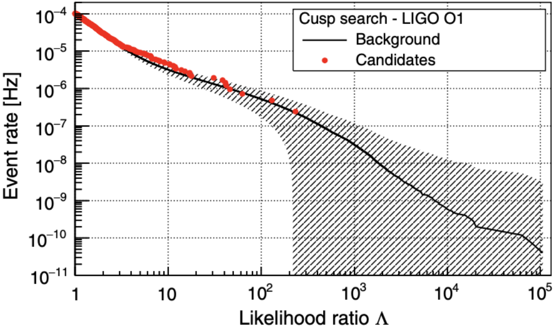
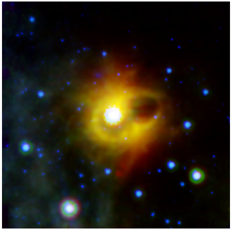


Table from Abbott+ PRD **97** 102002 (2018)

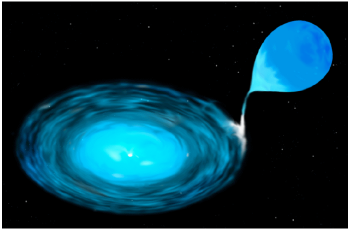
Neutron stars and oscillations

- Young pulsars **glitch**.
- Magnetars **flare**.
- Low-mass X-ray binaries **burst**.

Crab pulsar



SGR 1900+14



LMXB

- Which sorts of oscillation mode are relevant, to what amplitudes are they excited, and what are the GW/EM signitures?

Types of neutron star oscillation modes

- Neutron stars can *oscillate*, emitting GWs in the process.
- There exist a whole zoo of types of these normal modes:
  - f-mode: the fundamental mode (restoring force is pressure).
  - p-mode: overtones of fundamental (restoring force is pressure).
  - g-mode: restoring force due to composition/temperature gradients.
  - r-mode: restoring force in Coriolis force (only in rotating stars).
  - Alfvén mode: restoring force due to magnetic field.
  - Elastic mode: restoring force elastic.
- Key point is that frequency and lifetime of oscillation related to internal stellar structure.
- Hope to perform ‘gravitational wave asteroseismology’!

Basic equations

- Should work in General Relativity, but here's a Newtonian treatment, for simplicity.
- Fundamental equations relate pressure P, velocity $\mathbf{v}$, density ρ and gravitational potential Φ:
    - Newton's 2nd law:
    $$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho}\nabla P - \nabla\Phi + \mathbf{F}_{\text{other}}, \tag{1}$$
    - Poisson's equation:
    $$\nabla^2\Phi = 4\pi G\rho, \tag{2}$$
    - Conservation of mass:
    $$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0, \tag{3}$$

  where $\mathbf{F}_{\text{other}}$ contains any other forces you choose to model (viscosity, magnetic fields, elasticity, . . .)

Computing modes

- If unperturbed star is static and spherical, we can write

$$P_{\rm perturbed}(t, r, \theta, \phi) = P_{\rm unperturbed}(r) + \delta P(t, r, \theta, \phi). \tag{4}$$

- Can then substitute into the differential equations, and separate out the zero order (unperturbed) terms and the first order terms.
- Typically expand perturbed quantities assuming a harmonic time dependence, and in terms of spherical harmonics, e.g.

$$\delta P(t, r, \theta, \phi) = \sum_{lm} \delta P_{lm}(r) Y_{lm}(\theta, \phi) e^{i\omega_{lm}t}. \tag{5}$$

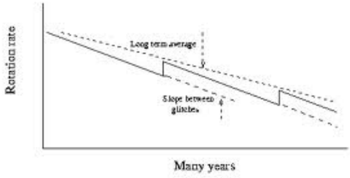
- Imposing the boundary conditions then leads to eigenmode solutions with discrete frequencies.

Simplest example: non-rotating uniform density star

- For non-rotating uniform density fluid star, can analytically find ‘Kelvin modes’, with frequency
$$\omega_{lm}^2 = \frac{4\pi G\rho}{3}\frac{2l(l-1)}{2l+1}. \tag{6}$$
- Writing in terms of $f_{lm} = \omega_{lm}/(2\pi)$ and parameterising:
$$f_{lm} \approx 3\,\mathrm{kHz}\sqrt{\frac{l(l-1)}{2l+1}}\left(\frac{M}{1.4M_\odot}\right)^{1/2}\left(\frac{10^6\,\mathrm{cm}}{R}\right)^{3/2}. \tag{7}$$
- e.g. for $l = 2$, get $f \approx 2$ kHz. This sets the frequency scale for GW for such modes—at the extreme high end of LIGO/Virgo.

Glitches

- Most of the time pulsar spin frequencies gradually decrease.
- Occasionally some younger pulsars undergo sudden spin-ups.



- Fractional variation in spin frequency small, e.g.

$$\frac{\Delta\Omega}{\Omega} \sim 10^{-6}$$

for Vela; this is considered a violent glitchier!

- Key question: do glitches generate GWs?

Glitch energies: a naive estimate

- In absence of detailed model, can make a 'naive' estimate:

$$E_{\rm glitch} = I\Omega\Delta\Omega = I\Omega^2 \frac{\Delta\Omega}{\Omega}.$$

- Parameterising with Vela in mind:

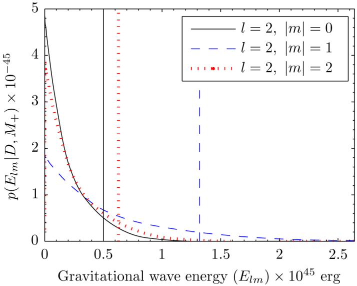
$$E_{\rm glitch} \sim 4.95 \times 10^{42}\,{\rm erg} \left(\frac{f_{\rm spin}}{11.2\,{\rm Hz}}\right)^2 \left(\frac{\Delta\Omega/\Omega}{10^{-6}}\right),$$

$$h_{\rm rss} \sim 10^{-22}\,{\rm Hz}^{1/2} \left(\frac{287\,{\rm pc}}{r}\right) \left(\frac{f_{\rm spin}}{11.2{\rm Hz}}\right) \left(\frac{1\,{\rm kHz}}{f_{\rm GW}}\right) \left(\frac{\Delta\Omega/\Omega}{10^{-6}}\right)^{1/2}.$$

- But can all this energy be put into modes, and if so which?

The Vela glitch paper: f-modes

- Search for ‘fundamental’ f-mode excitation following a Vela glitch was carried out (Abadie+ 2011).
- Looked for damped sinusoids with $f \sim$ 1–3 kHz, $\tau \lesssim$ 0.5 s.
- Found energy release $\Delta E_{\rm GW} \lesssim 10^{45}$ erg.



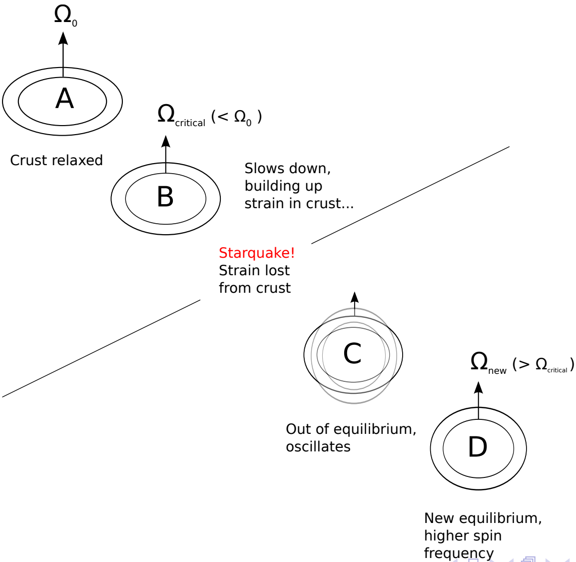
Do glitches excite oscillation modes?

There have been a small number of attempts to answer this question.

- van Eysden and Melatos (2008) looked at two-component infinitely long cylinder.
- Sidery, Passamonti, Andersson (2009) looked at two-component spherical spin-up using time evolutions.
- Keer & DIJ (2015) looked at the *starquake* model.

Starquakes & mode excitaiton

Keer & DIJ (2015) considered a specific evolutionary sequence:



Projecting the initial data onto the modes

Do indeed find that fundamental-like mode excited:

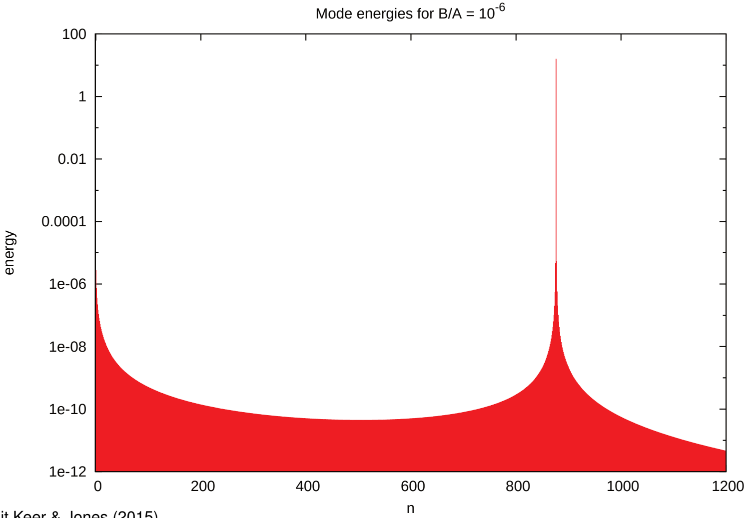


Figure credit Keer & Jones (2015)

Magnetars

- Magnetars are neutron stars with very strong magnetic fields, $B \sim 10^{15}$ G.
- Two sub-classes seem to occasionally emit violent outbursts of gamma rays:
    - Soft-gamma ray pulsars (SGRs)
    - Anomalous X-ray pulsars (AXPs)
- Believed to represent reconfigurations of the magnetic field, possibly triggered by crust cracking.
- Oscillations seen in the light curves of some bursts:

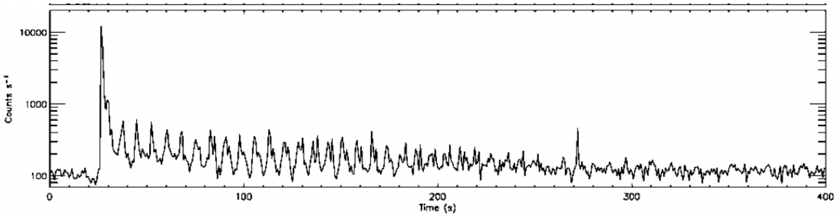
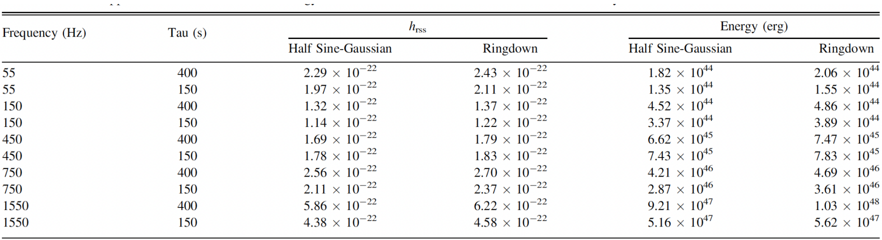


Figure credit Watts & Strohmayer (2006)

Magnetars

- Note clear what the excited modes are.
- Frequencies from tens of Hz to hundreds.
- Could be some combination of elastic and magnetic forces.
- Example: LVK searched over wide range for February 25 Burst from SGR 1806-20:



Frequency (Hz)	Tau (s)	$h_{\rm rss}$		Energy (erg)	
		Half Sine-Gaussian	Ringdown	Half Sine-Gaussian	Ringdown
55	400	2.29×10^{-22}	2.43×10^{-22}	1.82×10^{44}	2.06×10^{44}
55	150	1.97×10^{-22}	2.11×10^{-22}	1.35×10^{44}	1.55×10^{44}
150	400	1.32×10^{-22}	1.37×10^{-22}	4.52×10^{44}	4.86×10^{44}
150	150	1.14×10^{-22}	1.22×10^{-22}	3.37×10^{44}	3.89×10^{44}
450	400	1.69×10^{-22}	1.79×10^{-22}	6.62×10^{45}	7.47×10^{45}
450	150	1.78×10^{-22}	1.83×10^{-22}	7.43×10^{45}	7.83×10^{45}
750	400	2.56×10^{-22}	2.70×10^{-22}	4.21×10^{46}	4.69×10^{46}
750	150	2.11×10^{-22}	2.37×10^{-22}	2.87×10^{46}	3.61×10^{46}
1550	400	5.86×10^{-22}	6.22×10^{-22}	9.21×10^{47}	1.03×10^{48}
1550	150	4.38×10^{-22}	4.58×10^{-22}	5.16×10^{47}	5.62×10^{47}

Figure credit Abbott+ ApJ **874** 63 (2019)

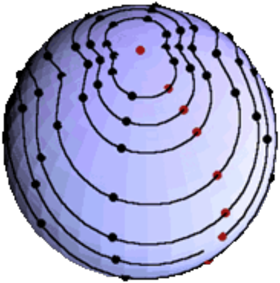
R-modes

- R-modes occur only in rotating stars; the restoring force is the Coriolis force.
- The GW frequency is proportional to the star's spin frequency:

$$f_{\mathrm{GW}} \approx \frac{4}{3} f_{\mathrm{spin}}. \tag{8}$$

- The r-modes are prime candidates for undergoing the *Chandrasekhar-Friedman-Schutz* (CFS) instability.

R-mode image: Hannah/Owen:



Growth of the instability

- Amplitude of a normal mode evolves as $\alpha(t) \sim e^{-t/\tau}$, where

$$\frac{1}{\tau} = \frac{1}{\tau_{\rm GRR}} + \frac{1}{\tau_{\rm damp}}.$$

- R-mode gravitational radiation reaction timescale scales sharply with $f_{\rm spin}$:

$$\tau_{\rm GRR} \approx -1 \text{ hour} \left(\frac{435 \text{ Hz}}{f_{\rm spin}}\right)^6 \frac{1}{M_{1.4} R_{11.7}^4},$$

  for an $n = 1$ polytrope (Owen et al 1998); weak dependence on equation of state.

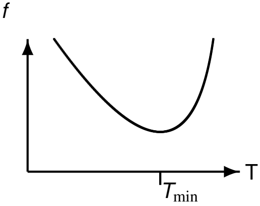
- Viscous damping timescale is temperature and spin frequency dependent:

$$\tau_{\rm damp} = \tau_{\rm damp}(f_{\rm spin}, T) > 0,$$

  strongly dependent on details of the equation of state.

Growth of the instability cont . . .

For given temperature, mode goes ‘CFS-unstable’ for $f_{\text{spin}} > f_{\text{CFS}}$:



- Newly born neurtron star may be very hot with fast spin.
- Can then cooling into instability window, and spin down rapidly.
- There is a large body of literature debating the details of this!

Summary

- There are many different sources for transient gravitational waves, including
  - Core collapse
  - Cosmic strings
  - Oscillating neurtron stars
- Frequency ranges of emission rather uncertain.
- For oscillations, frequency depends on sort of mode, and can be as high as several kHz.

Further reading

- Textbook: Shapiro & Teukolsky *Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects*, especially chapters 6, 9, 10.
- Textbook: Andersson *Gravitational-Wave Astronomy - Exploring the Dark Side of the Universe*, especially chapters 6, 13, 15, 18.
- Review article: Glampedakis & Gualtieri *Gravitational waves from single neutron stars: an advanced detector era survey*, arXiv 1709.07049.

Exercises

1. Use dimensional analysis to estimate the dynamical timescale for a self-gravitating star of mass M and radius R. Parameterise your result in units of $M = 1.4 M_\odot$ and $R = 10$ km. This sets the frequency scale for f-mode oscillation, and also the Keplerian (i.e. break-up) frequency, which in turn sets the maximum r-mode frequency.
2. Use dimensional analysis to estimate the timescale for oscillations in the elastic crust of a neutron star, with shear modulus μ, mass M and radius R. Parameterise your answer in terms of $\mu = 10^{30}$ erg cm^{-3}, $M = 1.4 M_\odot$ and $R = 10$ km. This sets the frequency for torsional crust oscillations.
3. Use dimensional analysis to estimate the timescale for *Alfvén* (i.e. magnetic) oscillations for a star with a magnetic field of strength B, mass M and radius R. Parameterise in units of $B = 10^{15}$ G, $M = 1.4 M_\odot$ and $R = 10$ km. [Hint: if you use Gaussian cgs units, then B^2 (with B in units of Gauss) is an energy density, with units erg cm^{-3}.